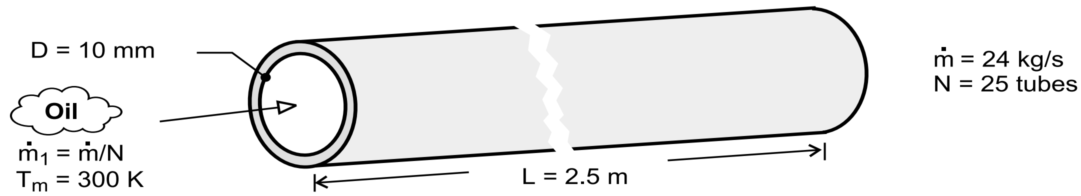
PROBLEM 8.5

KNOWN: Number, diameter and length of tubes and flow rate for an engine oil cooler.

FIND: Pressure drop and pump power (a) for flow rate of 24 kg/s and (b) as a function of flow rate for the range $10 \le \dot{m} \le 30$ kg/s.

SCHEMATIC:



ASSUMPTIONS: (1) Fully developed flow throughout the tubes.

PROPERTIES: *Table A.5*, Engine oil (300 K): $\rho = 884$ kg/m^3, $\mu = 0.486$ kg/s·m.

ANALYSIS: (a) Considering flow through a single tube, find

$$\mathrm{Re}_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4(24\,\mathrm{kg/s})}{25\pi(0.010\,\mathrm{m})0.486\,\mathrm{kg/s\cdot m}} = 251.5 \qquad (1)$$

Hence, the flow is laminar and from Equation 8.19,

$$f = \frac{64}{\mathrm{Re}_D} = \frac{64}{251.5} = 0.2545. \qquad (2)$$

With

$$u_m = \frac{\dot{m}_1}{\rho(\pi D^2/4)} = \frac{(25/25)\,\mathrm{kg/s}(4)}{(884\,\mathrm{kg/m^3})\pi(0.010\,\mathrm{m})^2} = 13.8\ \mathrm{m/s} \qquad (3)$$

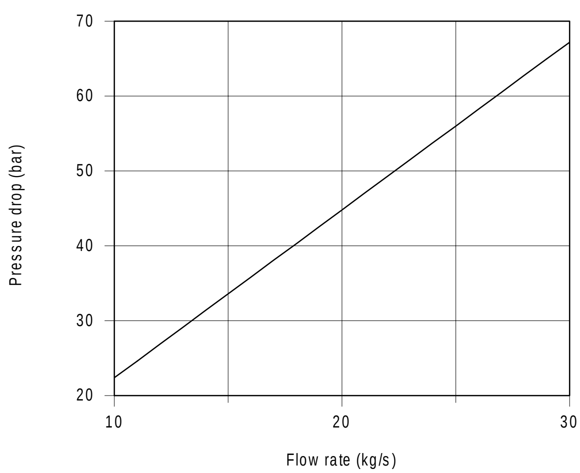
Equation 8.22a yields

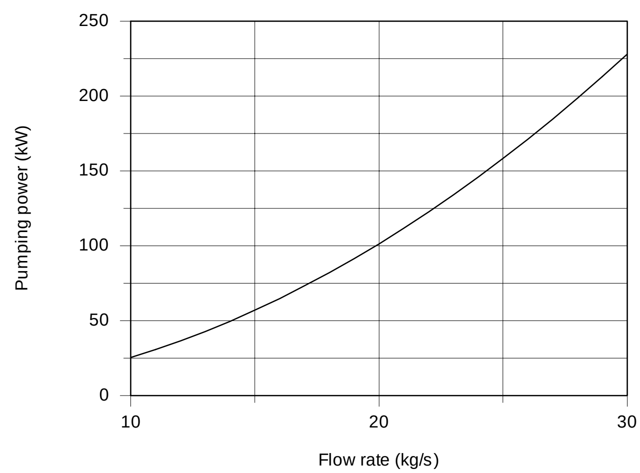
$$\Delta p = f\frac{\rho u_m^2}{2D}L = 0.2545\frac{(884\,\mathrm{kg/m^3})(13.8\,\mathrm{m/s})^2}{2(0.010\,\mathrm{m})}2.5\,\mathrm{m} = 5.38\times10^6\ \mathrm{N/m^2} = 53.8\ \mathrm{bar} \qquad (4)$$ <

The pump power requirement from Equation 8.23b,

$$P = \Delta p\cdot\dot{V} = \Delta p\cdot\frac{\dot{m}}{\rho} = 5.38\times10^6\ \mathrm{N/m^2}\ \frac{24\,\mathrm{kg/s}}{884\,\mathrm{kg/m^3}} = 1.459\times10^5\ \mathrm{N\cdot m/s} = 146\ \mathrm{W}. \qquad (5)$$ <

(b) Using IHT with the expressions of part (a), the pressure drop and pump power requirement as a function of flow rate, $\dot{m}$, for the range $10 \le \dot{m} \le 30$ kg/s are computed and plotted below.





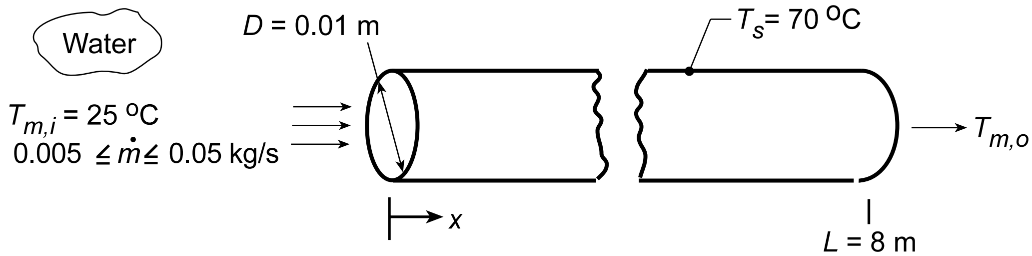
Continued...

PROBLEM 8.30

KNOWN: Diameter and length of copper tubing. Temperature of collector plate to which tubing is soldered. Water inlet temperature and flow rate.

FIND: (a) Water outlet temperature and heat rate, (b) Variation of outlet temperature and heat rate with flow rate. Variation of water temperature along tube for the smallest and largest flowrates.

SCHEMATIC:



ASSUMPTIONS: (1) Straight tube with smooth surface, (2) Negligible kinetic/potential energy and flow work changes, (3) Negligible thermal resistance between plate and tube inner surface, (4) $Re_{D,c}$ = 2300.

PROPERTIES: *Table A.6*, water (assume $\overline{T}_m = (T_{m,i} + T_s)/2 = 47.5°C = 320.5$ K): $\rho = 986$ kg/m^3, c_p = 4180 J/kg·K, $\mu = 577 \times 10^{-6}$ N·s/m^2, k = 0.640 W/m·K, Pr = 3.77. *Table A.6*, water (T_s = 343 K): μ_s = 400×10^{-6} N·s/m^2.

ANALYSIS: (a) For $\dot{m}$ = 0.01 kg/s, $Re_D = 4\,\dot{m}/\pi D\mu$ = 4(0.01 kg/s)/π(0.01 m)577 × 10^{-6} N·s/m^2 = 2200, in which case the flow may be assumed to be laminar. With $x_{fd,t}/D \approx 0.05 Re_D Pr$ = 0.05(2200)(3.77) = 415 and L/D = 800, the flow is fully developed over approximately 50% of the tube length. With $\left[Re_D Pr/(L/D)\right]^{1/3}(\mu/\mu_s)^{0.14}$ = 2.30, Eq. 8.57 may therefore be used to compute the average convection coefficient

$$\overline{Nu}_D = 1.86\left(\frac{Re_D\,Pr}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14} = 4.27$$

$$\overline{h} = (k/D)\overline{Nu}_D = 4.27(0.640\,W/m\cdot K)/0.01\,m = 273\,W/m^2\cdot K$$

From Eq. 8.42b,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{\pi DL}{\dot{m}c_p}\overline{h}\right) = \exp\left(-\frac{\pi \times 0.01\,m \times 8\,m \times 273\,W/m^2\cdot K}{0.01\,kg/s \times 4180\,J/kg\cdot K}\right)$$

$$T_{m,o} = T_s - 0.194\left(T_s - T_{m,i}\right) = 70°C - 8.7°C = 61.3°C$$ <

Hence, $q = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right) = 0.01\,kg/s\,(4186\,J/kg\cdot K)(36.3\,K) = 1519\,W$ <

(b) The IHT *Correlations*, *Rate Equations* and *Properties* Tool Pads were used to determine the parametric variations. The effect of $\dot{m}$ was considered in two steps, the first corresponding to $\dot{m}$ < 0.011 kg/s (Re_D < 2300) and the second for $\dot{m}$ > 0.011 kg/s (Re_D > 2300). In the first case, Eq. 8.57 was used to determine $\overline{h}$, while in the second Eq. 8.60 was used. The effects of $\dot{m}$ are as follows.

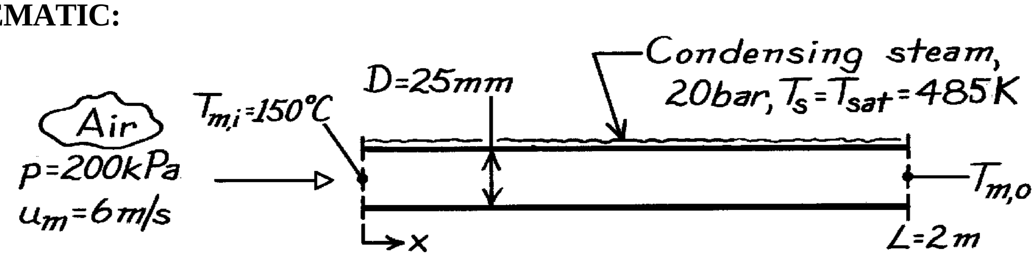
Continued...

PROBLEM 8.47

KNOWN: Air at prescribed inlet temperature and mean velocity heated by condensing steam on its outer surface.

FIND: (a) Air outlet temperature, pressure drop and heat transfer rate and (b) Effect on parameters of part (a) if pressure were doubled.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible kinetic and potential energy changes, (3) Thermal resistance of tube wall and condensate film are negligible.

PROPERTIES: *Table A-4*, Air (assume $\overline{T}_m$ = 450K, 1 atm = 101.3 kPa): $\rho = 0.7740\ \text{kg/m}^3$, $c_p = 1021$ J/kg·K, $\mu = 250.7 \times 10^{-7}\ \text{N·s/m}^2$, k = 0.0373 W/m·K, Pr = $\mu c_p/k$ = 0.686. Note that only ρ is pressure dependent; i.e., ρ α P; *Table A-6,* Saturated water (20 bar): $T_{sat} = T_s$ = 485K.

ANALYSIS: (a) For constant wall temperature heating, from Eq. 8.46 but with $U \approx \overline{h}_i$ since $\overline{h}_o >> \overline{h}_i$, where $\overline{h}_o$ is the convection coefficient for the condensing steam,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}\, c_p}\overline{h}_i\right)$$

where P = πD. For the air flow, find the mass rate and Reynolds number,

$$\dot{m} = r A_c u_m = 0.7740\ \text{kg/m}^3 (200\ \text{kPa}/101.3\ \text{kPa})\left(p\ (0.025\ \text{m})^2/4\right)\times 6\ \text{m/s}$$

$$\dot{m} = 4.501\times10^{-3}\text{kg/s}.$$

$$\text{Re}_D = \frac{4\dot{m}}{mp\ D} = \frac{4\times4.501\times10^{-3}\ \text{kg/s}}{250.7\times10^{-7}\ \text{N}\cdot\text{s/m}^2\times p\ (0.025\ \text{m})} = 9.143\times10^3.$$

Using the Dittus-Boelter correlation for fully-developed turbulent flow,

$$\text{Nu}_D = 0.023\text{Re}^{4/5}\ \text{Pr}^{0.4} = 0.023\left(9.143\times10^3\right)^{4/5}(0.682)^{0.4} = 29.12$$

$$h_i = \text{Nu}\cdot k/D = 29.12\times0.0373\ \text{W/m}\cdot\text{K}/0.025\ \text{m} = 43.4\ \text{W/m}^2\cdot\text{K}.$$

Hence, the outlet temperature is

$$\frac{212 - T_{m,o}}{(212-150)^\circ\text{C}} = \exp\left[-\frac{p\ (0.025\ \text{m})\times2\ \text{m}\times43.4\ \text{W/m}^2\cdot\text{K}}{4.501\times10^{-3}\ \text{kg/s}\times1021\ \text{J/kg}\cdot\text{K}}\right]$$

$$T_{m,o} = 198^\circ\text{C}.$$ <

Continued …..

The pressure drop follows from Eqs. 8.20 and 8.22,

$$f = 0.316 Re_D^{-1/4} = 0.316\left(9.143\times10^3\right)^{-1/4} = 0.0323$$

$$\Delta p = f\,\frac{r u_m^2}{2D} L$$

$$\Delta p = 0.0323 \frac{0.7740\ \text{kg/m}^3 (200/101.3)(6\ \text{m/s})^2 \times 2\ \text{m}}{2\times0.025\ \text{m}} = 71.1\ \text{N/m}^2.$$ <

The heat transfer rate is

$$q = \dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right) = 4.501\times10^{-3}\ \text{kg/s}\times1021\ \text{J/kg}\cdot\text{K}\,(198 - 150)\,\text{K} = 221\ \text{W}.$$ <

(b) If the pressure were doubled, we can see from the above relations, that $\dot{m}\ a\ r$, hence

$$\dot{m} = 2\dot{m}_o$$

$$Re_D = 2Re_{D,o},$$

since

$$h_i \propto (Re)^{4/5} \rightarrow \left(h_i / h_{i,o}\right) = 2^{4/5},$$

$$h_i = 1.74 h_{i,o}.$$

It follows that $T_{m,o}$ = 195°C, so that the effect on temperature is slight. However, the pressure drop increases by the factor $2(2)^{-1/4}$ = 1.68 and the heat rate by 2(195 - 150)/(198 - 150) = 1.88. In summary:

Parameter	p = 200 kPa Part (a)	p = 400 kPa Part (b)	Increase, %
$\dot{m}$, kg/s × 10^3	4.501	9.002	100
h_i, W/m^2·K	43.4	86.8	100
T_{mo} - $T_{m,i}$°C	48	45	-6
Δp, N/m^2	71.1	119	68
q, W	221	415	88

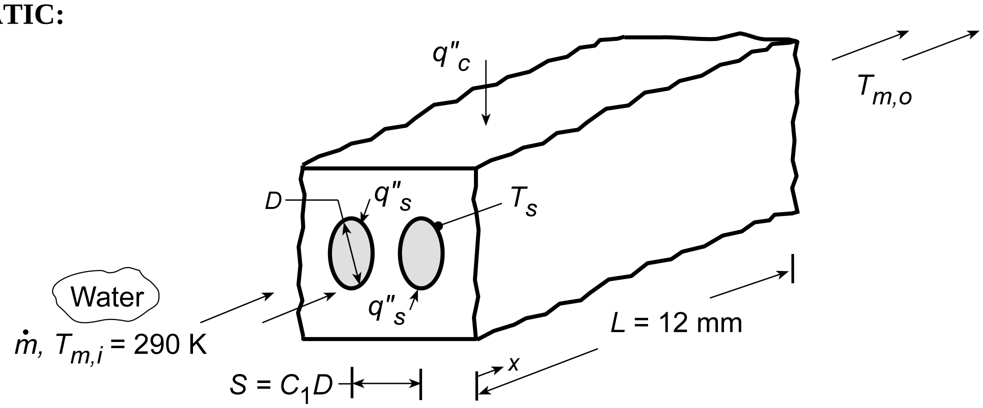
COMMENTS: (1) Note that $\overline{T}_m$ = (198 + 150)°C/2 = 447 K agrees well with the assumed value (450 K) used to evaluate the thermophysical properties.

PROBLEM 8.50

KNOWN: Configuration of microchannel heat sink.

FIND: (a) Expressions for longitudinal distributions of fluid mean and surface temperatures, (b) Coolant and channel surface temperature distributions for prescribed conditions, (c) Effect of heat sink design and operating conditions on the chip heat flux for a prescribed maximum allowable surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Negligible PE, KE and flow work changes, (3) All of the chip power dissipation is transferred to the coolant, with a uniform surface heat flux, q''_s, (4) Laminar, fully developed flow, (5) Constant properties.

PROPERTIES: *Table A.6*, Water (assume $\overline{T}_m = T_{m,i}$ = 290 K): c_p = 4184 J/kg·K, $\mu = 1080 \times 10^{-6}$ N·s/m^2, k = 0.598 W/m·K, Pr = 7.56.

ANALYSIS: (a) The number of channels passing through the heat sink is N = L/S = L/C_1D, and conservation of energy dictates that

$$q''_c L^2 = N(\pi DL)q''_s = \pi L^2 q''_s / C_1$$

which yields

$$q''_s = \frac{C_1 q''_c}{\pi} \tag{1}$$

With the mass flowrate per channel designated as $\dot{m}_1 = \dot{m}/N$, Eqs. 8.41 and 8.28 yield

$$T_m(x) = T_{m,i} + \frac{q''_s \pi D}{\dot{m}_1 c_p} x = T_{m,i} + \frac{L q''_c}{\dot{m} c_p} x \tag{2}$$ <

$$T_s(x) = T_m(x) + \frac{q''_s}{h} = T_m(x) + \frac{C_1 q''_c}{\pi h} \tag{3}$$ <

where, for laminar, fully developed flow with uniform q''_s, Eq. 8.53 yields h = 4.36 k/D.

(b) With L = 12 mm, D = 1 mm, C_1 = 2 and $\dot{m}$ = 0.01 kg/s, it follows that S = 2 mm, N = 6 and Re_D = $4\dot{m}_1/\pi D\mu = 4(0.01\,\text{kg/s})/6\pi(0.001\,\text{m})1.08\times10^{-3}\,\text{N}\cdot\text{s/m}^2$ = 1965. Hence, the flow is laminar, as assumed, and h = 4.36(0.598 W/m·K/0.001 m) = 2607 W/m^2·K. From Eqs. (2) and (3) the outlet mean and surface temperatures are

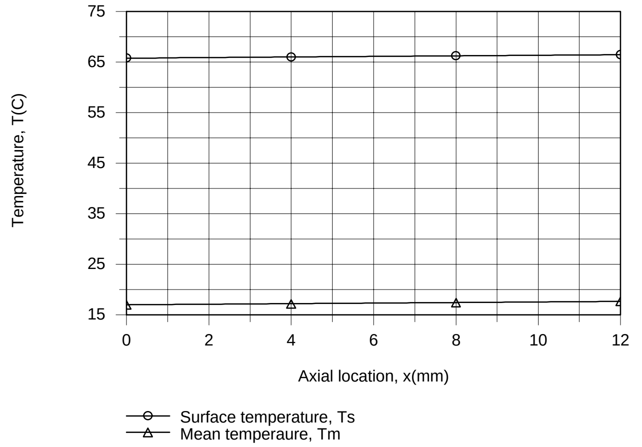
$$T_{m,o} = 290\,\text{K} + \frac{(0.012\,\text{m})^2\, 20\times10^4\,\text{W/m}^2}{0.01\,\text{kg/s}\,(4184\,\text{J/kg}\cdot\text{K})} = 290.7\,\text{K} = 17.7^\circ\text{C}$$

$$T_{s,o} = T_{m,o} + \frac{2}{\pi} \times \frac{20\times10^4\,\text{W/m}^2}{2607\,\text{W/m}^2\cdot\text{K}} = 339.5\,\text{K} = 66.5^\circ\text{C}$$

Continued...

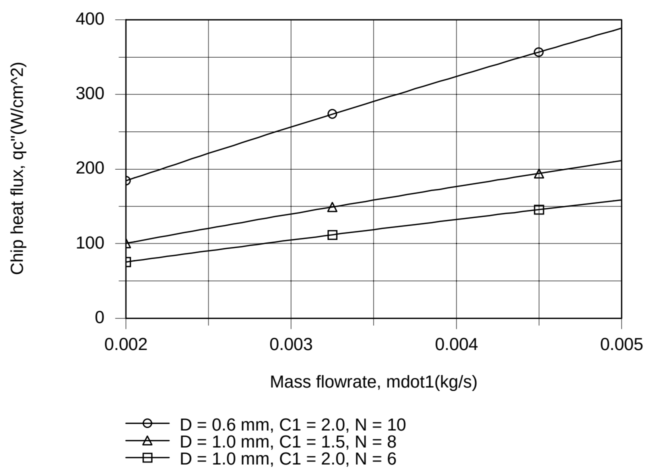
PROBLEM 8.50 (Cont.)

The axial temperature distributions are as follows



The flowrate is sufficiently large (and the convection coefficient sufficiently low) to render the increase in T_m and T_s with increasing x extremely small.

(c) The desired constraint of $T_{s,max} \le 50°C$ is not met by the foregoing conditions. An obvious and logical approach to achieving improved performance would involve increasing $\dot{m}_1$ such that turbulent flow is maintained in each channel. A value of $\dot{m}_1 > 0.002$ kg/s would provide $Re_D > 2300$ for D = 0.001. Using Eq. 8.60 with n = 0.4 to evaluate Nu_D and accessing the Correlations Toolpad of IHT to explore the effect of variations in $\dot{m}_1$ for different combinations of D and C_1, the following results were obtained.



We first note that a significant increase in q_c'' may be obtained by operating the channels in turbulent flow. In addition, there is an obvious advantage to reducing C_1, thereby increasing the number of channels for a fixed channel diameter. The biggest enhancement is associated with reducing the channel diameter, which significantly increases the convection coefficient, as well as the number of channels for fixed C_1. For $\dot{m}_1 = 0.005$ kg/s, h increases from 32,400 to 81,600 $W/m^2 \cdot K$ with decreasing D from 1.0 to 0.6 mm. However, for fixed $\dot{m}_1$, the mean velocity in a channel increases with decreasing D and care must be taken to maintain the flow pressure drop within acceptable limits.

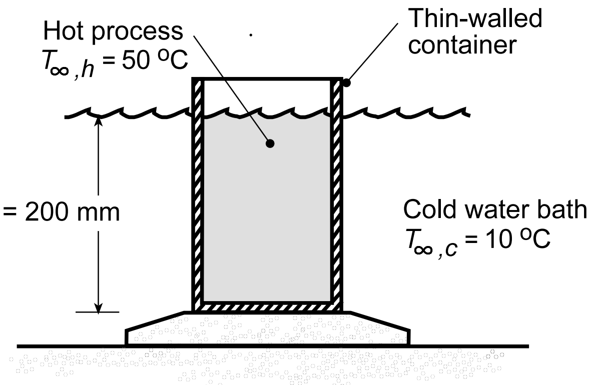
COMMENTS: Although the distribution computed for $T_m(x)$ in part (b) is correct, the distribution for $T_s(x)$ represents an upper limit to actual conditions due to the assumption of fully developed flow throughout the channel.

PROBLEM 9.22

KNOWN: Thin-walled container with hot process fluid at 50°C placed in a quiescent, cold water bath at 10°C.

FIND: (a) Overall heat transfer coefficient, U, between the hot and cold fluids, and (b) Compute and plot U as a function of the hot process fluid temperature for the range $20 \le T_{\infty,h} \le 50°C$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Heat transfer at the surfaces approximated by free convection from a vertical plate, (3) Fluids are extensive and quiescent, (4) Hot process fluid thermophysical properties approximated as those of water, and (5) Negligible container wall thermal resistance.

PROPERTIES: *Table A.6*, Water (assume $T_{f,h}$ = 310 K): $\rho_h = 1/1.007 \times 10^{-3} = 993$ kg/m^3, $c_{p,h} = 4178$ J/kg·K, $\nu_h = \mu_h/\rho_h = 695 \times 10^{-6}$ N·s/m^2/993 kg/m^3 = 6.999×10^{-7} m^2/s, $k_h = 0.628$ W/m·K, $Pr_h = 4.62$, $\alpha_h = k_h/\rho_h c_{p,h} = 1.514 \times 10^{-7}$ m^2/s, $\beta_h = 361.9 \times 10^{-6}$ K^{-1} ; *Table A.6*, Water (assume $T_{f,c}$ = 295 K): $\rho_c = 1/1.002 \times 10^{-3} = 998$ kg/m^3, $c_{p,c} = 4181$ J/kg·K, $\nu_c = \mu_c/\rho_c = 959 \times 10^{-6}$ N·s/m^2/998 kg/m^3 = 9.609×10^{-7} m^2/s, k_c = 0.606 W/m·K, $Pr_c = 6.62$, $\alpha_c = k_c/\rho_c c_{p,c} = 1.452\times 10^{-7}$ m^2/s, $\beta_c = 227.5 \times 10^{-6}$ K^{-1}.

ANALYSIS: (a) The overall heat transfer coefficient between the hot process fluid, $T_{\infty,h}$, and the cold water bath fluid, $T_{\infty,c}$, is

$$U = \left(1/\bar{h}_h + 1/\bar{h}_c\right)^{-1} \qquad (1)$$

where the average free convection coeffieicnts can be estimated from the vertical plate correlation Eq. 9.26, with the Rayleigh number, Eq. 9.25,

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{8/27}}\right\}^2 \qquad Ra_L = \frac{g\beta\Delta T L^3}{\nu\alpha} \qquad (2,3)$$

To affect a solution, assume $T_s = \left(T_{\infty,h} - T_{\infty,i}\right)/2 = 30^\circ C = 303\,K$, so that the hot and cold fluid film temperatures are $T_{f,h}$ = 313 K ≈ 310 K and $T_{f,c}$ = 293 K ≈ 295 K. From an energy balance across the container walls,

$$\bar{h}_h\left(T_{\infty,h} - T_s\right) = \bar{h}_c\left(T_s - T_{\infty,c}\right) \qquad (4)$$

the surface temperature T_s can be determined. Evaluating the correlation parameters, find:

Hot process fluid:

$$Ra_{L,h} = \frac{9.8\,m/s^2 \times 361.9\times10^{-6}\,K^{-1}\,(50-30)\,K\,(0.200m)^3}{6.999\times10^{-7}\,m^2/s \times 1.514\times10^{-7}\,m^2/s} = 5.357\times10^9$$

Continued...

PROBLEM 9.22 (Cont.)

$$\overline{Nu}_{L,h} = \left\{ 0.825 + \frac{0.387\left(5.357\times10^{9}\right)^{1/6}}{\left[1+\left(0.492/4.62\right)^{9/16}\right]^{8/27}} \right\}^{2} = 251.5$$

$$\bar{h}_h = \overline{Nu}_{L,h}\,\frac{h_h}{L} = 251.5\times0.628\,\mathrm{W/m^2\cdot K}/0.200\,\mathrm{m} = 790\,\mathrm{W/m^2\cdot K}$$

Cold water bath:

$$Ra_{L,c} = \frac{9.8\,\mathrm{m/s^2}\times227.5\times10^{-6}\,\mathrm{K^{-1}}\,(30-10)\,\mathrm{K}\,(0.200\,\mathrm{m})^3}{9.609\times10^{-7}\,\mathrm{m^2/s}\times1.452\times10^{-7}\,\mathrm{m^2/s}} = 2.557\times10^{9}$$

$$\overline{Nu}_{L,c} = \left\{ 0.825 + \frac{0.387\left(2.557\times10^{9}\right)^{1/6}}{\left[1+\left(0.492/6.62\right)^{9/16}\right]^{8/27}} \right\}^{2} = 203.9$$

$$\bar{h}_c = 203.9\times0.606\,\mathrm{W/m\cdot K}/0.200\,\mathrm{m} = 618\,\mathrm{W/m^2\cdot K}$$

From Eq. (1) find

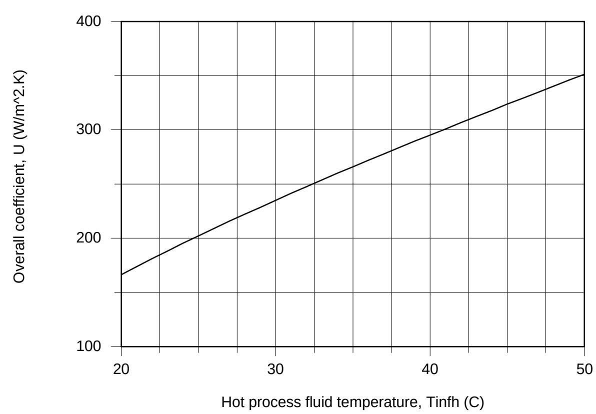
$$U = \left(1/790 + 1/618\right)^{-1}\,\mathrm{W/m^2\cdot K} = 347\,\mathrm{W/m^2\cdot K}$$ <

Using Eq.(4), find the resulting surface temperature

$$790\,\mathrm{W/m^2\cdot K}\,(50-T_s)\,\mathrm{K} = 618\,\mathrm{W/m^2\cdot K}\,(T_s-30)\,\mathrm{K} \qquad T_s = 32.4^\circ\mathrm{C}$$

Which compares favorably with our assumed value of 30°C.

(b) Using the *IHT Correlations Tool, Free Convection, Vertical Plate* and following the foregoing approach, the overall coefficient was computed as a function of the hot fluid temperature and is plotted below. Note that U increases almost linearly with $T_{\infty,h}$.



COMMENTS: For the conditions of part (a), using the IHT model of part (b) with thermophysical properties evaluated at the proper film temperatures, find U = 352 W/m·K with T_s = 32.4°C. Our approximate solution was a good one.

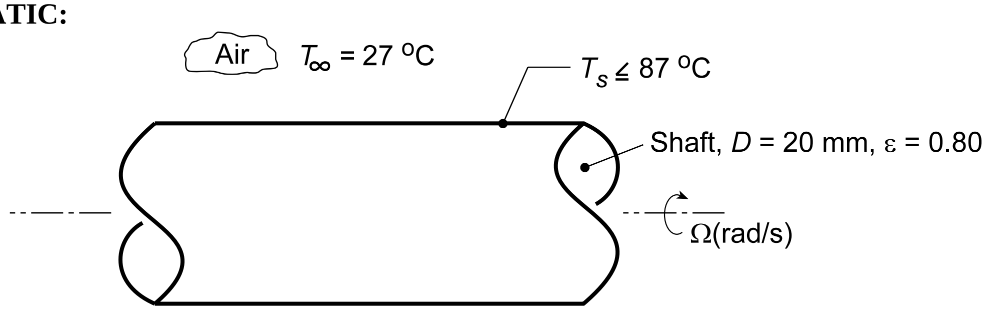
(2) Because the set of equations for part (b) is quite stiff, when using the IHT model you should follow the suggestions in the IHT Example 9.2 including use of the intrinsic function Tfluid_avg (T1,T2).

PROBLEM 9.63

KNOWN: Motor shaft of 20-mm diameter operating in ambient air at $T_\infty = 27°C$ with surface temperature $T_s \le 87°C$.

FIND: Convection coefficients and/or heat removal rates for different heat transfer processes: (a) For a rotating horizontal cylinder as a function of rotational speed 5000 to 15,000 rpm using the recommended correlation, (b) For free convection from a horizontal stationary shaft; investigate whether mixed free and forced convection effects for the range of rotational speeds in part (a) are significant using the recommended criterion; (c) For radiation exchange between the shaft having an emissivity of 0.8 and the surroundings also at ambient temperature, $T_{sur} = T_\infty$; and (d) For cross flow of ambient air over the stationary shaft, required air velocities to remove the heat rates determined in part (a).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Shaft is horizontal with isothermal surface.

PROPERTIES: *Table A.4*, Air ($T_f = (T_s + T_\infty)/2 = 330$ K, 1 atm): $\nu = 18.91 \times 10^{-6}$ m²/s , k = 0.02852 W/m·K, $\alpha = 26.94 \times 10^{-6}$ m²/s, Pr = 0.7028, $\beta = 1/T_f$.

ANALYSIS: (a) The recommended correlation for the a horizontal rotating shaft is

$$\overline{Nu}_D = 0.133 Re_D^{2/3} Pr^{1/3} \qquad Re_D < 4.3 \times 10^5 \qquad 0.7 < Pr < 670$$

where the Reynolds number is

$$Re_D = \Omega D^2 / \nu$$

and Ω (rad/s) is the rotational velocity. Evaluating properties at $T_f = (T_s + T_\infty)/2$, find for $\omega = 5000$ rpm,

$$Re_D = (5000 \text{rpm} \times 2\pi \text{ rad/rev} / 60 \text{s/min})(0.020\text{m})^2 / 18.91 \times 10^{-6} \text{ m}^2/\text{s} = 11,076$$

$$\overline{Nu}_D = 0.133(11,076)^{2/3}(0.7028)^{1/3} = 58.75$$

$$\bar{h}_{D,rot} = \overline{Nu}_D\, k/D = 58.75 \times 0.02852 \text{ W/m} \cdot \text{K}/0.020\text{m} = 83.8 \text{ W/m}^2 \cdot \text{K} \qquad <$$

The heat rate per unit shaft length is

$$q'_{rot} = \bar{h}_{D,rot}(\pi D)(T_s - T_\infty) = 83.8 \text{ W/m}^2 \cdot \text{K}(\pi \times 0.020\text{m})(87-27)^\circ \text{C} = 316 \text{ W/m} \qquad <$$

The convection coefficient and heat rate as a function of rotational speed are shown in a plot below.

(b) For the stationary shaft condition, the free convection coefficient can be estimated from the Churchill-Chu correlation, Eq. (9.34) with

Continued...

PROBLEM 9.63 (Cont.)

$$\mathrm{Ra_D} = \frac{g\beta\Delta T D^3}{\nu\alpha}$$

$$\mathrm{Ra_D} = \frac{9.8\,\mathrm{m/s^2}\,(1/330\mathrm{K})(87-27)\mathrm{K}\,(0.020\mathrm{m})^3}{18.91\times10^{-6}\,\mathrm{m^2/s}\times26.94\times10^{-6}\,\mathrm{m^2/s}} = 27{,}981$$

$$\overline{\mathrm{Nu}}_\mathrm{D} = \left\{0.60 + \frac{0.387\mathrm{Ra}_\mathrm{D}^{1/6}}{\left[1+(0.559/\mathrm{Pr})^{9/16}\right]^{8/27}}\right\}^2$$

$$\overline{\mathrm{Nu}}_\mathrm{D} = \left\{0.60 + \frac{0.387\,(27{,}981)^{1/6}}{\left[1+(0.559/0.7028)^{9/16}\right]^{8/27}}\right\}^2 = 5.61$$

$$\bar{h}_{D,fc} = \overline{\mathrm{Nu}}_\mathrm{D}\,k/D = 5.61\times0.02852\,\mathrm{W/m\cdot K}/0.020\mathrm{m} = 8.00\,\mathrm{W/m^2\cdot K}$$

$$q'_{fc} = 8.00\,\mathrm{W/m^2\cdot K}\,(\pi\times0.020\mathrm{m})(87-27)^\circ\,\mathrm{C} = 30.2\,\mathrm{W/m} \qquad <$$

Mixed free and forced convection effects may be significant if

$$\mathrm{Re_D} < 4.7\left(\mathrm{Gr}_\mathrm{D}^3/\mathrm{Pr}\right)^{0.137}$$

where $\mathrm{Gr_D} = \mathrm{Ra_D}/\mathrm{Pr}$, find using results from above and in part (a) for ω = 5000 rpm,

$$11{,}076 \;\;?<?\;\; 4.7\left[(27{,}981/0.7028)^3/0.7018\right]^{0.137} = 383$$

We conclude that free convection effects are not significant for rotational speeds above 5000 rpm.

(c) Considering radiation exchange between the shaft and the surroundings,

$$h_{rad} = \varepsilon\sigma\,(T_s + T_{sur})\left(T_s^2 + T_{sur}^4\right)$$

$$h_{rad} = 0.8\times5.67\times10^{-8}\,\mathrm{W/m^2\cdot K}\,(360+300)\left(360^2+300^2\right)\mathrm{K}^3 = 6.57\,\mathrm{W/m^2\cdot K} \qquad <$$

and the heat rate by radiation exchange is

$$q'_{rad} = h_{rad}\,(\pi D)(T_s - T_{sur})$$

$$q'_{rad} = 6.57\,\mathrm{W/m^2\cdot K}\,(\pi\times0.020\mathrm{m})(87-27)\mathrm{K} = 24.8\,\mathrm{W/m} \qquad <$$

(d) For cross flow of ambient air at a velocity V over the shaft, the convection coefficient can be estimated using the Churchill-Bernstein correlation, Eq. 7.57, with

$$\mathrm{Re}_{D,cf} = \frac{VD}{\nu}$$

$$\overline{\mathrm{Nu}}_{D,cf} = \bar{h}_{D,cf}\,D/k = 0.3 + \frac{0.62\,\mathrm{Re}_{D,cf}^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+(0.4/\mathrm{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_{D,cf}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

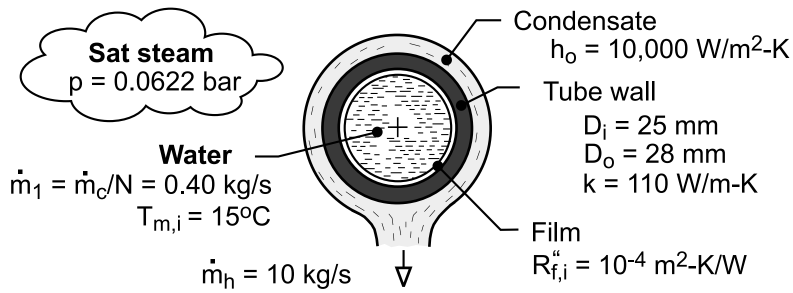
Continued...

PROBLEM 11.7

KNOWN: Number, inner-and outer diameters, and thermal conductivity of condenser tubes. Convection coefficient at outer surface. Overall flow rate, inlet temperature and properties of water flow through the tubes. Flow rate and pressure of condensing steam. Fouling factor for inner surface.

FIND: (a) Overall coefficient based on outer surface area, U_o, without fouling, (b) Overall coefficient with fouling, (c) Temperature of water leaving the condenser.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible flow work and kinetic and potential energy changes for water flow, (2) Fully-developed flow in tubes, (3) Negligible effect of fouling on D_i.

PROPERTIES: Water (Given): $c_p = 4180$ J/kg·K, $\mu = 9.6 \times 10^{-4}$ N·s/m^2, k = 0.60 W/m·K, Pr = 6.6. *Table A-6*, Water, saturated vapor (p = 0.0622 bars): $T_{sat} = 310$ K, $h_{fg} = 2.414 \times 10^6$ J/kg.

ANALYSIS: (a) Without fouling, Eq. 11.5 yields

$$\frac{1}{U_o} = \frac{1}{h_i}\left(\frac{D_o}{D_i}\right) + \frac{D_o \ln(Do/Di)}{2k_t} + \frac{1}{h_o}$$

With $Re_{D_i} = 4\dot{m}_1/\pi D_i \mu = 1.60\,kg/s/\left(\pi \times 0.025m \times 9.6 \times 10^{-4}\ N \cdot s/m^2\right) = 21,220,$ flow in the tubes is turbulent, and from Eq. 8.60

$$h_i = \left(\frac{k}{D_i}\right)0.023 Re_{D_i}^{4/5} Pr^{0.4} = \left(\frac{0.60\,W/m\cdot K}{0.025m}\right)0.023(21,200)^{4/5}(6.6)^{0.4} = 3400\,W/m^2\cdot K$$

$$U_o = \left[\frac{1}{3400}\left(\frac{28}{25}\right) + \frac{0.028\ln(28/25)}{2\times 110} + \frac{1}{10,000}\right]^{-1} W/m^2\cdot K =$$

$$\left(3.29\times 10^{-4} + 1.44\times 10^{-5} + 10^{-4}\right)^{-1} W/m^2\cdot K = 2255\,W/m^2\cdot K \qquad <$$

(b) With fouling, Eq. 11.5 yields

$$U_o = \left[4.43\times 10^{-4} + (Do/Di)R''_{f,i}\right]^{-1} = \left(5.55\times 10^{-4}\right)^{-1} = 1800\,W/m^2\cdot K \qquad <$$

(c) The rate at which energy is extracted from the steam equals the rate of heat transfer to the water, $\dot{m}_h h_{fg} = \dot{m}_c c_p \left(T_{m,o} - T_{m,i}\right)$, in which case

$$T_{m,o} = T_{m,i} + \frac{\dot{m}_h h_{fg}}{\dot{m}_c c_p} = 15°C + \frac{10\,kg/s \times 2.414\times 10^6\,J/kg}{400\,kg/s \times 4180\,J/kg\cdot K} = 29.4°C \qquad <$$

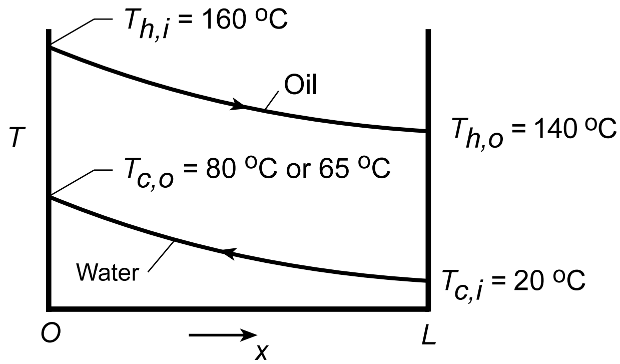
COMMENTS: (1) The largest contribution to the thermal resistance is due to convection at the interior of the tube. To increase U_o, hi could be increased by increasing $\dot{m}_1$, either by increasing $\dot{m}_c$ or decreasing N. (2) Note that $T_{m,o} = 302.4$ K < $T_{sat} = 310$ K, as must be the case.

PROBLEM 11.16

KNOWN: Inner tube diameter (D = 0.02 m) and fluid inlet and outlet temperatures corresponding to design conditions for a concentric tube heat exchanger. Overall heat transfer coefficient (U = 500 $W/m^2 \cdot K$) and desired heat rate (q = 3000 W). Cold fluid outlet temperature after three years of operation.

FIND: (a) Required heat exchanger length, (b) Heat rate, hot fluid outlet temperature, overall heat transfer coefficient, and fouling factor after three years.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to the surroundings and kinetic and potential energy changes, (2) Negligible tube wall conduction resistance, (3) Constant properties.

ANALYSIS: (a) The tube length needed to achieve the prescribed conditions may be obtained from Eqs. 11.14 and 11.15 where $\Delta T_1 = T_{h,i} - T_{c,o} = 80°C$ and $\Delta T_2 = T_{h,o} - T_{c,i} = 120°C$. Hence, $\Delta T_{1m} = (120 - 80)°C/\ln(120/80) = 98.7°C$ and

$$L = \frac{q}{(\pi D) U \Delta T_{1m}} = \frac{3000\,W}{(\pi \times 0.02\,m) 500\,W/m^2 \cdot K \times 98.7°C} = 0.968\,m$$ <

(b) With $q = C_c(T_{c,o} - T_{c,i})$, the following ratio may be formed in terms of the design and 3 year conditions.

$$\frac{q}{q_3} = \frac{C_c\left(T_{c,o} - T_{c,i}\right)}{C_c\left(T_{c,o} - T_{c,i}\right)_3} = \frac{60°C}{45°C} = 1.333$$

Hence,

$$q_3 = q/1.33 = 3000\,W/1.333 = 2250\,W$$ <

Having determined the ratio of heat rates, it follows that

$$\frac{q}{q_3} = \frac{C_h\left(T_{h,i} - T_{h,o}\right)}{C_h\left(T_{h,i} - T_{h,o}\right)_3} = \frac{20°C}{160°C - T_{h,o(3)}} = 1.333$$

Hence,

$$T_{h,o(3)} = 160°C - 20°C/1.333 = 145°C$$ <

With $\Delta T_{lm,3} = (125 - 95)/\ln(125/95) = 109.3°C$,

$$U_3 = \frac{q_3}{(\pi DL)\Delta T_{lm,3}} = \frac{2250\,W}{\pi(0.02\,m)0.968\,m(109.3°C)} = 338\,W/m^2 \cdot K$$ <

Continued...

PROBLEM 11.16 (Cont.)

With $U = \left[(1/h_i) + (1/h_o)\right]^{-1}$ and $U_3 = \left[(1/h_i) + (1/h_o) + R''_{f,c}\right]^{-1}$,

$$R''_{f,c} = \frac{1}{U_3} - \frac{1}{U} = \left(\frac{1}{338} - \frac{1}{500}\right) m^2 \cdot K/W = 9.59 \times 10^{-4} m^2 \cdot K/W \quad <$$

COMMENTS: Over time fouling will always contribute to a degradation of heat exchanger performance. In practice it is desirable to remove fluid contaminants and to implement a regular maintenance (cleaning) procedure.

PROBLEM 11.32

KNOWN: Single pass, cross-flow heat exchanger with hot exhaust gases (mixed) to heat water (unmixed)

FIND: Required surface area.

SCHEMATIC:

ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible kinetic and potential energy changes, (3) Exhaust gas properties assumed to be those of air.

PROPERTIES: *Table A-6*, Water ($\overline{T}_c$ = (80 + 30)°C/2 = 328 K): c_p = 4184 J/kg·K; *Table A-4*, Air (1 atm, $\overline{T}_h$ = (100 + 225)°C/2 = 436 K): c_p = 1019 J/kg·K.

ANALYSIS: The rate equation for the heat exchanger follows from Eqs. 11.14 and 11.18. The area is given as

$$A = q/U\Delta T_{\ell m} = q/UF\Delta T_{\ell m,CF} \tag{1}$$

where F is determined from Fig. 11.13 using

$$P = \frac{80-30}{225-30} = 0.26 \text{ and } R = \frac{225-100}{80-30} = 2.50 \text{ giving } F \approx 0.92. \tag{2}$$

From an energy balance on the cold fluid, find

$$q = \dot{m}_c c_c \left(T_{c,o} - T_{c,i}\right) = 3\frac{\text{kg}}{\text{s}} \times 4184\frac{\text{J}}{\text{kg}\cdot\text{K}}(80-30)\text{K} = 627{,}600 \text{ W}. \tag{3}$$

From Eq. 11.15, the LMTD for counter-flow conditions is

$$\Delta T_{\ell m,CF} = \frac{\Delta T_1 - \Delta T_2}{\ell n\left(\Delta T_1/\Delta T_2\right)} = \frac{(225-80)-(100-30)}{\ell n(145/70)}\text{°C} = 103.0\text{°C}. \tag{4}$$

Substituting numerical values resulting from Eqs. (2-4) into Eq. (1), find the required surface area to be

$$A = 627{,}600\,\text{W}/200\,\text{W/m}^2\cdot\text{K} \times 0.92 \times 103.0\text{K} = 33.1\text{m}^2.$$ <

COMMENTS: Note that the properties of the exhaust gases were not needed in this method of analysis. If the ε-NTU method were used, find first C_h/C_c = 0.40 with $C_{min} = C_h$ = 5021 W/K. From Eqs. 11.19 and 11.20, with $C_h = C_{min}$, $\varepsilon = q/q_{max} = (T_{h,i} - T_{h,o})/(T_{h,i} - T_{c,i})$ = (225 – 100)/(225 – 30) = 0.64. Using Fig. 11.19 with C_{min}/C_{max} = 0.4 and ε = 0.64, find NTU = UA/C_{min} ≈ 1.4. Hence,

$$A = NTU \cdot C_{min}/U \approx 1.4 \times 5021\,\text{W/K}/200\,\text{W/m}^2\cdot\text{K} = 35.2\text{m}^2.$$

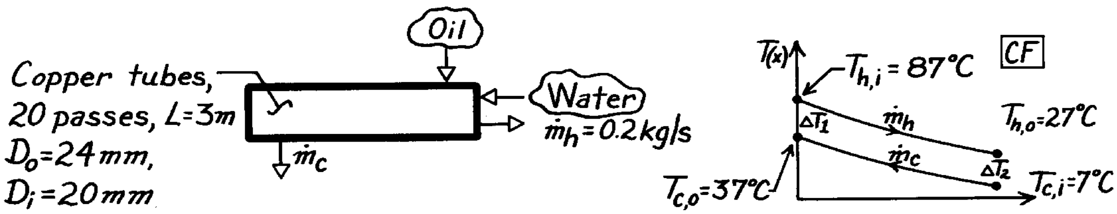
Note agreement with above result.

PROBLEM 11.52

KNOWN: Shell-and-tube heat exchanger with one shell pass and 20 tube passes.

FIND: Average convection coefficient for the outer tube surface.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Negligible changes in kinetic and potential energies, (3) Constant properties, (4) Type of oil not specified, (5) Thermal resistance of tubes negligible; no fouling.

PROPERTIES: *Table A-6*, Water, liquid ($\overline{T}_h$ = 330 K): c_p = 4184 J/kg·K, k = 0.650 W/m·K, μ = 489 $\times 10^{-6}$ N·s/m^2, Pr = 3.15.

ANALYSIS: To find the average coefficient for the outer tube surface, h_o, we need to evaluate h_i for the internal tube flow and U, the overall coefficient. From Eq. 11.5,

$$\frac{1}{UA}=\frac{1}{h_iA_i}+\frac{1}{h_oA_o}=\frac{1}{N_t\pi L}\left[\frac{1}{h_iD_i}+\frac{1}{h_oD_o}\right]$$

where N_t is the total number of tubes. Solving for h_o,

$$h_o=D_o^{-1}\left[(UA)^{-1}N_t\pi L-1/h_iD_i\right]^{-1}. \tag{1}$$

Evaluate h_i from an appropriate correlation; begin by calculating the Reynolds number.

$$Re_{D,i}=\frac{4\,\dot{m}_h}{\pi D_i\mu}=\frac{4\times 0.2\ \text{kg/s}}{\pi\,(0.020\text{m})\,489\times10^{-6}\,\text{N}\cdot\text{s/m}^2}=26{,}038.$$

Hence, flow is turbulent and since L >> D_i, the flow is likely to be fully developed. Use the Dittus-Boelter correlation with n = 0.3 since $T_s < T_m$, $Nu_D = 0.023\ Re_D^{4/5}\ Pr^{0.3}$

$$h_i=\frac{k}{D}Nu_D=\frac{0.650\ \text{W/m}\cdot\text{K}}{0.020\text{m}}\times0.023(26{,}038)^{4/5}(3.15)^{0.3}=3594\ \text{W/m}^2\cdot\text{K}. \tag{2}$$

To evaluate UA, we need to employ the rate equation, written as

$$UA=q/F\Delta T_{\ell n,CF} \tag{3}$$

where q = $\dot{m}_h\ c_{p,h}\ (T_{h,i}-T_{h,o})$ = 0.2 kg/s × 4184 J/kg·K) (87-27)°C = 50,208 W and $\Delta T_{\ell n,CF}$ = $[\Delta T_1 - \Delta T_2]/\ell n\ (\Delta T_1/\Delta T_2)$ = [(87 – 37) – (27 – 7)]°C/ ℓn (87 – 37/27 – 7) = 32.7°C. Find F ≈ 0.5 using Fig. 11.10 with P = (27 – 87)/(7 – 87) = 0.75 and R = (7 – 37)/(27 – 87) = 0.50. Substituting numerical values in Eqs. (3) and (1), find

$$UA=50{,}208\,\text{W}/0.5\times32.7°\text{C}=3071\,\text{W/K} \tag{4}$$

$$h_o=(0.024\text{m})^{-1}\left[(3071\ \text{W/K})^{-1}\times20\times\pi\times3\text{m}-1/3594\ \text{W/m}^2\cdot\text{K}\times0.020\text{m}\right]^{-1}=878\ \text{W/m}^2\cdot\text{K}. \quad <$$

COMMENTS: Using the ε-NTU method: find C_h and C_c to obtain C_r = 0.5 and ε = 0.75. From Eq. 11.31b,c find NTU = 3.59 and UA = 3003 W/K.